Supporting information

The instantaneous frequency of a signal can be interpreted as the rate of change of phase, and is expressed as Hz according to the following equation:

\[ f(x) = \frac{1}{2\pi} \frac{d}{dt} \phi(t) \]

where \( \Phi(t) \) represents the arg function of a real-valued signal \( s(t) \), that can be unwrapped to an unconstrained interval representing a continuous function of argument \( t \) in the form:

\[ \phi(t) = 2\pi \int_{-\infty}^{t} f(\tau)d\tau \]

Granger causality is based on the assumption that a past sample of a series \( x(n) \) contributes significantly to the prediction of \( y(n) \). Assessing the extent of Granger causality provides a measure of the strength of interaction among structures based on the rationale that predictable variations in a series - due to the consistent emergence of variations in another series \( b \) - takes place if their mechanisms of generation are somehow physically linked, such as through active neuroanatomic connections. Some statistical Granger causality tests (GCTs) are based on the direct examination of \( a_{ij}(r) \) coefficients describing the linear prediction effect of the \( r \)-th past sample \( x_j(n-r) \) of \( x_i(n) \) on predicting \( x_i(n) \). Thus, if statistical analysis can show that \( a_{ij}(r) = 0 \) for all values of \( r \), then the hypothesis of \( x_j(n) \) Granger-causing \( x_i(n) \) can be rejected. The frequency domain equivalent of this concept is \( \tilde{a}_{ij}(f) = 0 \) for all \( f \). Thus, after suitable normalization, the definition of partial directed coherence becomes:

\[ \pi_{ij}(f) \Delta a_{ij}(f) \]

where \( \tilde{a}_{ij} \) is the complex conjugate.

Taken alone, \( \pi_{ij}(f) \) describes the linear pairwise time series relatedness, without reference to the direction of information flow. The main advantage of partial directed coherence is that it allows Granger causality to be assessed over specific frequency ranges, in contrast to the Granger causality test, which considers the spectrum as a whole.